

Section 1.3: Surface Areas of Objects Made from Right Rectangular Prisms

Surface Area – the total area of all the surfaces (faces) of an object.

Consider the following:

Assume that the area of each face of this cube is 1 unit².



1. What is the surface area of 1 cube? 6 units²
2. What is the surface area of 2 cubes? 12 units²
3. What is the surface area of 3 cubes? 18 units²
4. Put two cubes together to make a train. What is the surface area of the train? 10 units²

5. Continue to place cubes at the end of the train. Complete the table:

Number of Cubes	Surface Area (units ²)
1	6
2	10
3	14
4	18
5	22

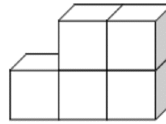
6. What happens to the surface area each time you place another cube in the train?

The surface area will increase by 4 units².

7. Why is there a difference between your answer in #2 and #4?

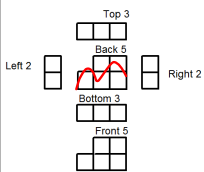
There is a difference because when the cubes form a train, an overlap occurs.

Find the surface area of the linking cubes below:



There are two methods for finding the area of linking cubes.

Method 1. Count the squares on all 6 views of the object.



- | | |
|-----------|----------|
| 1. Top | 4. Back |
| 2. Bottom | 5. Left |
| 3. Front | 6. Right |

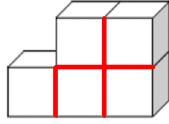
Surface Area = $3 + 3 + 5 + 5 + 2 + 2 = 20$ units².

Method 2. Count the total square faces on all the cubes and subtract the overlap.

There are 5 cubes each with 6 faces.

Therefore, $5 \times 6 = 30$ units².

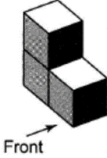
Subtract 2 faces for each place that the squares are joined, or overlap.
There are 5 places that are joined.



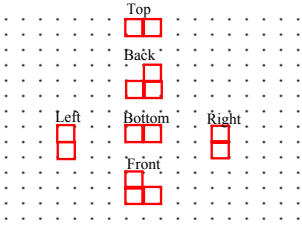
Therefore, $5 \times 2 = 10$ units² and there are 10 overlapping faces.

Surface Area = 30 - 10 = 20 units².

Determine the surface area of each composite object:


1. 

Top
Back
Left
Bottom
Right
Front



Surface Area = $2 + 3 + 2 + 3 + 2 + 2 = 14$ units².

There are 3 cubes each with 6 faces.
 Therefore, $3 \times 6 = 18$ units² total.

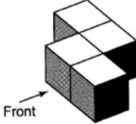
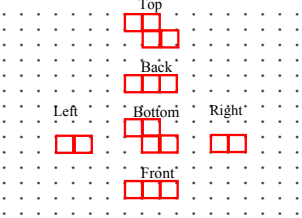


There are 2 places that are joined.

Therefore, $2 \times 2 = 4$ units² and there are 4 overlapping faces.


Surface Area = $18 - 4 = 14$ units².

2.

Surface Area = $4 + 3 + 4 + 3 + 2 + 2 = 18$ units².

There are 4 cubes each with 6 faces.
 Therefore, $4 \times 6 = 24$ units² total.

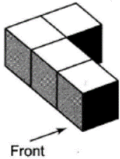
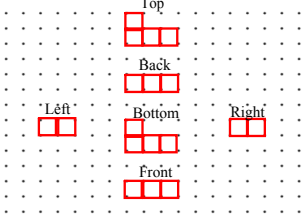


There are 3 places that are joined.

Therefore, $3 \times 2 = 6$ units² and there are 6 overlapping faces.

Surface Area = $24 - 6 = 18$ units².

3.


Surface Area = $4 + 3 + 4 + 3 + 2 + 2 = 18$ units².

There are 4 cubes each with 6 faces.
 Therefore, $4 \times 6 = 24$ units² total.

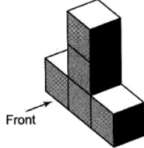
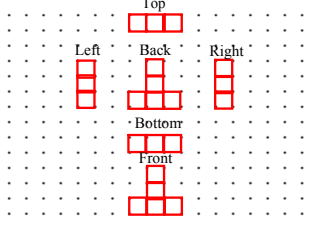
There are 3 places that are joined.

Therefore, $3 \times 2 = 6$ units² and there are 6 overlapping faces.

Surface Area = $24 - 6 = 18$ units².



4.


Surface Area = $3 + 5 + 3 + 5 + 3 + 3 = 22$ units².

There are 5 cubes each with 6 faces.
 Therefore, $5 \times 6 = 30$ units² total.

There are 4 places that are joined.

Therefore, $4 \times 2 = 8$ units² and there are 8 overlapping faces.

Surface Area = $30 - 8 = 22$ units².



Practice Exercises: p. 30 #4 b, c, e, f
 (use both methods)

Calculating the Surface Area of Square/Rectangular Prism

Recall: In a square/rectangular prism:

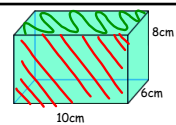
1. Opposite sides are congruent
2. The area of a square/rectangle can be found using

$$\text{Area} = \text{length} \times \text{width}$$

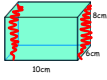
Complete the following:

1. Front/Back = length x width
= 10 cm x 8 cm
= 80 cm² each

Top/Bottom = length x width
= 10 cm x 6 cm
= 60 cm² each



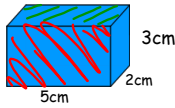
Left Side/Right Side = length x width
 = 8 cm x 6 cm
 = 48 cm² each



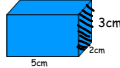
Total Surface Area = 2(80 cm²) + 2(60 cm²) + 2(48 cm²)
 = 160 cm² + 120 cm² + 96 cm²
 = 376 cm²

2. Front/Back = length x width
 = 5 cm x 3 cm
 = 15 cm² each

Top/Bottom = length x width
 = 5 cm x 2 cm
 = 10 cm² each



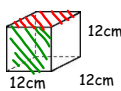
Left Side/Right Side = length x width
 $A = l \times w$
 $= 2 \times 3$
 $= 6 \text{ cm}^2$
 = 2 cm x 3 cm
 = 6 cm² each



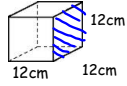
Total Surface Area = 2(15 cm²) + 2(10 cm²) + 2(6 cm²)
 = 30 cm² + 20 cm² + 12 cm²
 = 62 cm²

3. Front/Back = length x width
 = 12 cm x 12 cm
 = 144 cm² each

Top/Bottom = length x width
 = 12 cm x 12 cm
 = 144 cm² each



Left Side/Right Side = length x width
 = 12 cm x 12 cm
 = 144 cm² each



Total Surface Area = 2(144 cm²) + 2(144 cm²) + 2(144 cm²)
 = 288 cm² + 288 cm² + 288 cm²
 = 864 cm²

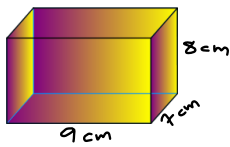
Is there a shorter method for square prisms ?

Since all sides are the same, you have 6 sides with the same area.

Each Side = length x width
 = 12 cm x 12 cm
 = 144 cm²

Total Surface Area = 6 x (Side Length)
 = 6 (144 cm²)
 = 864 cm²

Calculate the surface area of the following rectangular prism:

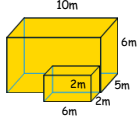


Finding the Surface Area of a Composite Object

A composite object is made from 2 or more objects.

To find the surface area of a composite object, imagine dipping the object in paint. The surface area is the area of all the faces covered in paint. The overlap is not painted. Therefore, the overlap is not part of the surface area.

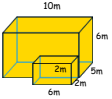
A. Step 1. Calculate the surface area of the larger prism.



Front/Back = length x width
 = 10 m x 6 m
 = 60 m² each

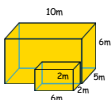
Top/Bottom = length x width
 = 10 m x 5 m
 = 50 m² each

Left Side/Right Side = length x width
 = 6 m x 5 m
 = 30 m² each



Total Surface Area = 2(60 m²) + 2(50 m²) + 2(30 m²)
 = 120 m² + 100 m² + 60 m²
 = 280 m²

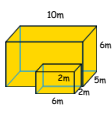
Step 2. Calculate the surface area of the smaller prism.



Front/Back = length x width
 = 6 m x 2 m
 = 12 m² each

Top/Bottom = length x width
 = 6 m x 2 m
 = 12 m² each

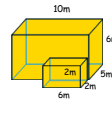
Left Side/Right Side = length x width
 = 2 m x 2 m
 = 4 m² each



Total Surface Area = 2(12 m²) + 2(12 m²) + 2(4 m²)
 = 24 m² + 24 m² + 8 m²
 = 56 m²

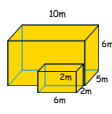
Step 3. Calculate the overlap.

Overlap = length x width
 = 6 m x 2 m
 = 12 m²



Step 4. Calculate the surface area of the composite object.

Surface Area of Composite Object
 = SA large prism + SA small prism - 2(overlap)
 = 280 m² + 56 m² - 2(12 m²)
 = 280 m² + 56 m² - 24 m²
 = 312 m²



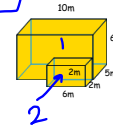
$TSA = SA_1 + SA_2 - \text{Overlaps}$

$SA_1 = 2lw + 2lh + 2wh$
 $= 2(10)(5) + 2(10)(6) + 2(5)(6)$
 $= 100 + 120 + 60$
 $= 280 \text{ m}^2$

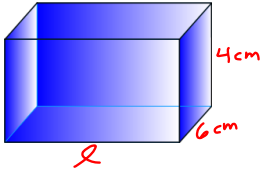
$SA_2 = 2lw + 2lh + 2wh$
 $= 2(6)(2) + 2(6)(2) + 2(2)(2)$
 $= 24 + 24 + 8$
 $= 56 \text{ m}^2$

Overlap: $A_{\square} = l \times w = 6 \times 2 = 12 \text{ m}^2$

$\therefore TSA = SA_1 + SA_2 - 2 \times \text{Overlap}$
 $= 280 + 56 - 2(12)$
 $= 280 + 56 - 24$
 $= 312 \text{ m}^2$



Challenge: Solve for the length

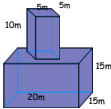


$SA = 208 \text{ cm}^2$


B. Step 1. Calculate the surface area of the larger prism.

Front/Back = length x width
 = 20 m x 15 m
 = 300 m² each

Top/Bottom = length x width
 = 20 m x 15 m
 = 300 m² each



Left Side/Right Side = length x width
 = 15 m x 15 m
 = 225 m² each

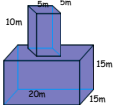


Total Surface Area = $2(300 \text{ m}^2) + 2(300 \text{ m}^2) + 2(225 \text{ m}^2)$
 = $600 \text{ m}^2 + 600 \text{ m}^2 + 450 \text{ m}^2$
 = 1650 m^2

Step 2. Calculate the surface area of the smaller prism.

Front/Back = length x width
 = 10 m x 5 m
 = 50 m² each

Top/Bottom = length x width
 = 5 m x 5 m
 = 25 m² each



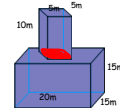
Left Side/Right Side = length x width
 = 10 m x 5 m
 = 50 m² each



Total Surface Area = 2(50 m²) + 2(25 m²) + 2(50 m²)
 = 100 m² + 50 m² + 100 m²
 = 250 m²

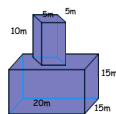
Step 3. Calculate the overlap.

Overlap = length x width
 = 5 m x 5 m
 = 25 m²



Step 4. Calculate the surface area of the composite object.

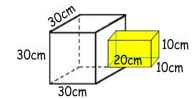
Surface Area of Composite Object
 = SA large prism + SA small prism - 2(overlap)
 = 1650 m² + 250 m² - 2(25 m²)
 = 1650 m² + 250 m² - 50 m²
 = 1850 m²



C. Step 1. Calculate the surface area of the larger prism.

Each Side = length x width
 = 30 cm x 30 cm
 = 900 cm²

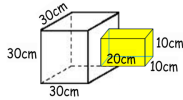
Total Surface Area = 6 x (Side Length)
 = 6 (900 cm²)
 = 5400 cm²



Step 2. Calculate the surface area of the smaller prism.

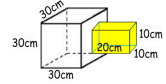
Front/Back = length x width
 = 20 cm x 10 cm
 = 200 cm² each

Top/Bottom = length x width
 = 20 cm x 10 cm
 = 200 cm² each



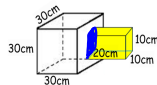
Left Side/Right Side = length x width
 = 10 cm x 10 cm
 = 100 cm² each

Total Surface Area = 2(200 cm²) + 2(200 cm²) + 2(100 cm²)
 = 400 cm² + 400 cm² + 200 cm²
 = 1000 cm²



Step 3. Calculate the overlap.

Overlap = length x width
 = 10 cm x 10 cm
 = 100 cm²

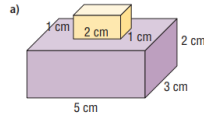


Step 4. Calculate the surface area of the composite object.

Surface Area of Composite Object

= SA large prism + SA small prism - 2(overlap)
 = 5400 cm² + 1000 cm² - 2(100 cm²)
 = 5400 cm² + 1000 cm² - 200 cm²
 = 6200 cm²

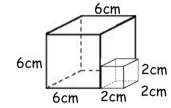
Calculate the SA of the comp. object P931



D. Step 1. Calculate the surface area of the larger prism.

$$\begin{aligned} \text{Each Side} &= \text{length} \times \text{width} \\ &= 6 \text{ cm} \times 6 \text{ cm} \\ &= 36 \text{ cm}^2 \end{aligned}$$

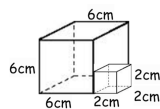
$$\begin{aligned} \text{Total Surface Area} &= 6 \times (\text{Side Length}) \\ &= 6 \times (36 \text{ cm}^2) \\ &= 216 \text{ cm}^2 \end{aligned}$$



Step 2. Calculate the surface area of the smaller prism.

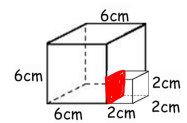
$$\begin{aligned} \text{Each Side} &= \text{length} \times \text{width} \\ &= 2 \text{ cm} \times 2 \text{ cm} \\ &= 4 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total Surface Area} &= 6 \times (\text{Side Length}) \\ &= 6 \times (4 \text{ cm}^2) \\ &= 24 \text{ cm}^2 \end{aligned}$$



Step 3. Calculate the overlap.

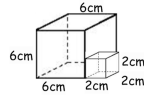
$$\begin{aligned} \text{Overlap} &= \text{length} \times \text{width} \\ &= 2 \text{ cm} \times 2 \text{ cm} \\ &= 4 \text{ cm}^2 \end{aligned}$$



Step 4. Calculate the surface area of the composite object.

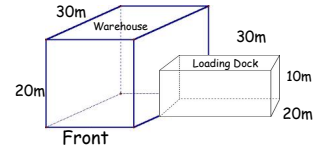
Surface Area of Composite Object

$$\begin{aligned}
 &= SA \text{ large prism} + SA \text{ small prism} - 2(\text{overlap}) \\
 &= 216 \text{ cm}^2 + 24 \text{ cm}^2 - 2(4 \text{ cm}^2) \\
 &= 216 \text{ cm}^2 + 24 \text{ cm}^2 - 8 \text{ cm}^2 \\
 &= 232 \text{ cm}^2
 \end{aligned}$$



E. A loading dock is attached to one wall of a warehouse. The exterior of the buildings is to be painted at a cost of \$2.50/m². How much will it cost to paint the buildings?

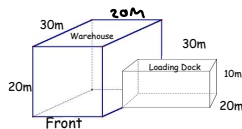
Will the bottom of the warehouse and loading dock be painted? No



Step 1. Calculate the surface area of the warehouse to be painted.

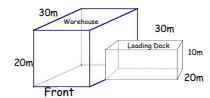
Front/Back = length x width
 = 20 m x 20 m
 = 400 m² each

Top/Bottom = length x width
 = 20 m x 30 m
 = 600 m² each



Left Side/Right Side = length x width
 = 20 m x 30 m
 = 600 m² each

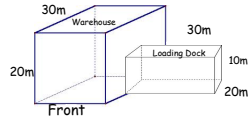
Total Surface Area = 2(400 m²) + 2(600 m²) + 2(600 m²)
 = 800 m² + 1200 m² + 1200 m²
 = 3200 m²



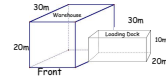
Step 2. Calculate the surface area of the loading dock to be painted.

Front/Back = length x width
 = 30 m x 10 m
 = 300 m² each

Top/Bottom = length x width
 = 30 m x 20 m
 = 600 m²



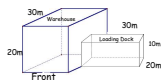
Left Side/Right Side = length x width
 = 20 m x 10 m
 = 200 m² each



Total Surface Area = 2(300 m²) + 2(600 m²) + 2(200 m²)
 = 600 m² + 1200 m² + 400 m²
 = 2200 m²

Step 3. Calculate the overlap.

Overlap = length x width
 = 20 m x 10 m
 = 200 m²



Step 4. Calculate the surface area of the composite object.

Surface Area of Composite Object
 = SA large prism + SA small prism - 2(overlap) - Roofs - Ground
 = 3200 m² + 2200 m² - 2(200 m²) - 2(600 m²) - 2(600 m²)
 = 3200 + 2200 - 400 - 1200 - 1200
 = 2600 m²

Cost = 2600 x \$2.50
 = \$6500

Practice Exercises: p. 31 #8a & p.46 #16a,b.