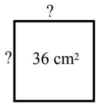
Section 1.2: Square Roots of Non-Perfect Squares

Recall:

1. If the area of a square is 36 cm^2 , what is the side length of the square?

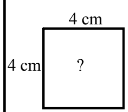


Side Length = $\sqrt{\text{Area}}$

= $\sqrt{36 \text{ cm}^2}$

= 6 cm

2. If the side length of a square is 4 cm, what is the area of the square?

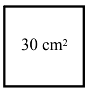


Area = $(\text{Side Length})^2$

= 4 cm x 4 cm

= 16 cm^2

If the area of a square is 30 cm^2 , what is the side length of the square?



Side Length = $\sqrt{\text{Area}}$

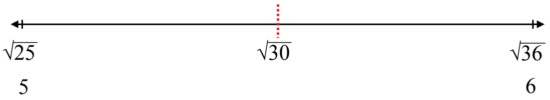
= $\sqrt{30 \text{ cm}^2}$

= ?

This is not a perfect square so we are unable to get an exact answer. However, we can estimate using the perfect squares.

Between which two consecutive perfect squares does 30 fall ?

Imagine a number line and use the knowledge that $\sqrt{25} = 5$ and $\sqrt{36} = 6$. Therefore, $\sqrt{30}$ must lie between 5 and 6.



$\sqrt{30}$ falls approximately half way between 5 and 6. Let's guess and check:

$(5.4)^2 = 5.4 \times 5.4 = 29.16$

$(5.5)^2 = 5.5 \times 5.5 = 30.25$

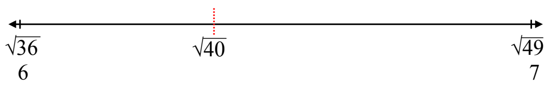
$(5.6)^2 = 5.6 \times 5.6 = 31.36$

The closest estimate is 5.5, therefore $\sqrt{30} \approx 5.5$.

Note: ≈ means approximately

Using guess and check, approximate $\sqrt{40}$.

Using a number line and the knowledge that $\sqrt{36} = 6$ and $\sqrt{49} = 7$, $\sqrt{40}$ must lie between 6 and 7.



Using guess and check:

$(6.2)^2 = 6.2 \times 6.2 = 38.44$

$(6.3)^2 = 6.3 \times 6.3 = 39.69$

$(6.4)^2 = 6.4 \times 6.4 = 40.96$

The closest estimate is 6.3. Therefore $\sqrt{40} \approx 6.3$.

Estimating the Square Root of a Non-Perfect Square Fraction

Two Methods:

1. Using a calculator (good for multiple choice)
2. Benchmark method (required for long answer)

Method 1: Calculator

Example. Use a calculator to estimate the square root of $\frac{15}{27}$.

$\sqrt{\frac{15}{27}} = \sqrt{(15 \div 27)}$

$= \sqrt{0.5}$

≈ 0.75

Method 2: Benchmarks

1. Find the perfect squares closest to the numerator and denominator, and write a new fraction using these numbers.

2. Take the square root of the new fraction. The square root of the new fraction will be approximately equal to the square root of the original fraction.

Examples. Use benchmarks to estimate the square root of:

A. $\frac{15}{27}$ Replace $\frac{15}{27}$ with $\frac{16}{25}$.

$$\sqrt{\frac{15}{27}} \approx \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

B. $\frac{8}{5}$ Replace $\frac{8}{5}$ with $\frac{9}{4}$.

$$\sqrt{\frac{8}{5}} \approx \sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$$

Practice Exercises: p. 18-19 #6, 12

Estimating the Square Root of a Non-Perfect Square Decimal

Two Methods:

1. Using a calculator
2. Benchmark method

Method 1: Calculator

Example. Use a calculator to estimate the square root of 58.9 to the nearest tenth.

$\sqrt{58.9} \approx 7.674 \approx 7.7$

Method 2: Benchmarks

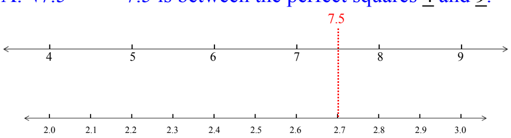
1. Look at the number under the square root sign. Find the closest perfect square above this number, and the closest perfect square below it.

2. Draw a number line containing the number under the square root sign and the closest perfect squares above and below. Be sure to use a ruler and measure off your values properly.

3. Using a ruler, draw another line below the previous line. Be sure the square roots are underneath the appropriate perfect square. Use these to estimate the square root of the non-perfect square number that was given.

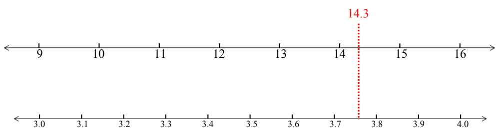
Example. Use benchmarks to estimate:

A. $\sqrt{7.5}$ 7.5 is between the perfect squares 4 and 9.



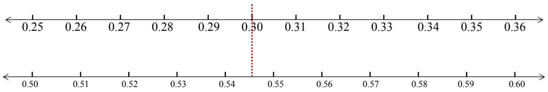
$\sqrt{7.5} \approx 2.7$

B. $\sqrt{14.3}$ 14.3 is between the perfect squares 9 and 16.



$\sqrt{14.3} \approx 3.76$

C. $\sqrt{0.30}$ 0.30 is between the perfect squares <u>0.25</u> and <u>0.36</u>.



$\sqrt{0.30} \approx 0.547$

Practice Exercises: p. 18-19 #7a,b,d, 9 & 16a

Finding a Number with a Square Root Between Two Given Numbers

To find a number with a square root between two given numbers:

1. Pick a number in between the two given values.
2. Square the number that you picked.

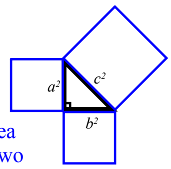
<u>Example.</u> Identify a decimal that has a square root between 10 and 11.

$(10.2)^2$ = 10.2 x 10.2 = 104.04

Practice Exercise: p.21, #11

Applications: Pythagorean Theorem

Recall: Pythagorean Theorem is a rule which states that, for any right triangle, the area of the square on the hypotenuse is equal to the sum of the area of the squares on the other two sides (legs).

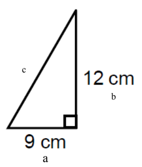


$a^2 + b^2 = c^2$

where, a and b are legs (sides) of the triangle and c is the hypotenuse.

Example. Solve for the length of the hypotenuse:

A.



$a^2 + b^2 = c^2$

$9^2 + 12^2 = c^2$

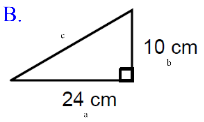
$81 + 144 = c^2$

$225 = c^2$

$c = \sqrt{225}$

$c = 15$ cm

B.



$a^2 + b^2 = c^2$

$24^2 + 10^2 = c^2$

$576 + 100 = c^2$

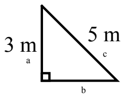
$676 = c^2$

$c = \sqrt{676}$

$c = 26$ cm

Example. Solve for the missing side lengths:

A.



$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

$9 + b^2 = 25$

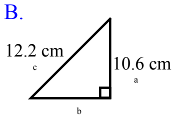
$9 - 9 + b^2 = 25 - 9$

$b^2 = 16$

$b = \sqrt{16}$

$b = 4$ m

B.



$a^2 + b^2 = c^2$

$(10.6)^2 + b^2 = (12.2)^2$

$112.36 + b^2 = 148.84$

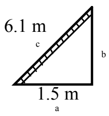
$112.36 - 112.36 + b^2 = 148.84 - 112.36$

$b^2 = 36.48$

$b = \sqrt{36.48}$

$b = 6.04$ cm

C. A ladder is 6.1 m long. The distance from the base of the ladder to the wall is 1.5 m. How far up the wall will the ladder reach?



$a^2 + b^2 = c^2$

$(1.5)^2 + b^2 = (6.1)^2$

$2.25 + b^2 = 37.21$

$2.25 - 2.25 + b^2 = 37.21 - 2.25$

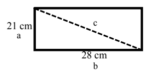
$b^2 = 34.96$

$b = \sqrt{34.96}$

$b = 5.9$

The ladder will reach 5.9 m up the wall.

D. The dimensions of a computer monitor are 28 cm by 21 cm. What is the length of the diagonal?



$a^2 + b^2 = c^2$

$21^2 + 28^2 = c^2$

$441 + 784 = c^2$

$1225 = c^2$

$c = \sqrt{1225}$

$c = 35$

The diagonal is 35 cm.

Practice Exercises: p. 19 #10,13