Section 1.1: Square Roots of Perfect Squares

Perfect squares can be written as fractions or decimals.

Part 1: Fractions

There are two ways to determine the square root of a perfect square that is written as a fraction:

1. Algebraically
2. Using a diagram (square)

<u>Algebraic Method:</u>

1. If the number is written as a mixed number, change it to an improper fraction.

2. Reduce the fraction to lowest terms (if possible).

3. Take the square root of the numerator and denominator.

Algebraically determine the square root of the following fractions:

A. $\dfrac{4}{9}$

$$\sqrt{\dfrac{4}{9}} = \dfrac{\sqrt{4}}{\sqrt{9}} = \dfrac{2}{3}$$

B. $4\frac{21}{25}$ * It is necessary to change the mixed number to an improper fraction

$$4\frac{21}{25} = \frac{121}{25}$$

$$\sqrt{\frac{121}{25}} = \frac{\sqrt{121}}{\sqrt{25}} = \frac{11}{5}$$

C. $\frac{8}{50}$ *It is necessary to reduce the fraction

$$\frac{8^{\div2}}{50^{\div2}} = \frac{4}{25}$$

$$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$$

Note: If we are asked to determine the square root of a perfect square fraction, and are expected to write the answer as a decimal, we can simply use our calculator to determine the answer.

Determine the square root of $\frac{4}{9}$ in decimal form:

Method 1:

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

*We can take the square root of the fraction, and then turn the resulting fraction into a decimal.

$$\frac{2}{3} = 2 \div 3 = 0.\overline{6}$$

Method 2:

$$\frac{4}{9} = 4 \div 9 = 0.\overline{4}$$

* We can change the fraction into a decimal, and then take the square root of the decimal.

$$\sqrt{0.\overline{4}} = 0.\overline{6}$$

Diagram Method:

1. Draw a larger square made of smaller squares (the number of smaller squares will be the denominator of the fraction).
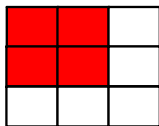
2. Starting at the top left corner of the larger square, shade in the number of squares equal to the numerator.

3. Determine the square root by writing the shaded length over the side length.
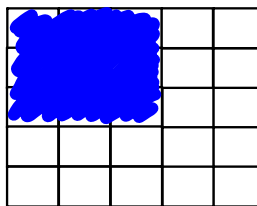
Use a diagram to determine the square root of:

A. $\dfrac{4}{9}$

B. $\dfrac{9}{25}$

Practice Exercises: Pg.11, #'s 3b,c,
5e,f,g,h & 7a,b,c,d

Part 2: Decimals

If we are asked to find the square root of a perfect square decimal number, we may either have to write the answer in decimal form or as a fraction.

Determine the square root of 1.5625 in decimal form using a calculator.

$$\sqrt{1.5625} = 1.25$$

If we are expected to write the square root of a perfect square decimal number in fraction form, we must first change the decimal number to a fraction.

How to Change a Decimal to a Fraction (Review)

A. 0.6

The 6 is in the first decimal position called the tenths place. Therefore,

$$0.6 = \frac{6}{10} \overset{\div 2}{\underset{\div 2}{}} = \frac{3}{5}$$

B. 0.08

The 8 is in the second decimal position called the hundredths place. Therefore,

$$0.08 = \frac{8}{100} \overset{\div 4}{\underset{\div 4}{}} = \frac{2}{25}$$

C. 0.25

The 5 is in the hundredths place. Therefore,

$$0.25 = \frac{25}{100} \overset{\div 25}{\underset{\div 25}{}} = \frac{1}{4}$$

Always look at the last number and that's the decimal position we are looking for!

Once we rewrite the decimal as a fraction, we can find the square root the same way we did for fractions.

Determine the square root of the following. Write the answers as a fraction:

A. $0.25 = \frac{25}{100} \overset{\div 25}{\underset{\div 25}{}} = \frac{1}{4}$

$\sqrt{0.25} = \sqrt{\frac{1}{4}}$

$= \frac{\sqrt{1}}{\sqrt{4}}$

$= \frac{1}{2}$

B. $0.09 = \frac{9}{100}$

$\sqrt{0.09} = \sqrt{\frac{9}{100}}$

$= \frac{\sqrt{9}}{\sqrt{100}}$

$= \frac{3}{10}$

To calculate a number whose square root is given, square the square root of the number to get the value (multiply the number by itself)

Calculate the number whose square root is:

A. $4.5: 4.5 \times 4.5 = 20.25$

B. $\dfrac{4}{5}: \dfrac{4}{5} \times \dfrac{4}{5} = \dfrac{16}{25}$

Practice Exercises: Pg.11, #'s 3a, 5a,b,c,d 7f,g,h,i & 9a,b,e,f

Determining Whether Given Numbers are Perfect Squares

All of the numbers that we have attempted to take the square root of so far have been perfect squares. However, not all numbers are perfect squares. Here, we will learn to determine whether or not fractions and decimals are perfect squares.

Part 1: Fractions

To determine whether a fraction is a perfect square, follow the process that we used earlier to determine the perfect square of the fraction. If the fraction is reduced and we CANNOT take the square root of both the numerator and denominator the fraction is NOT a perfect square.

Determine if the following are perfect squares:

A. $\dfrac{16}{19}$

Since 19 is not a perfect square, $\dfrac{16}{19}$ is not a perfect square.

B. $2\frac{2}{3} = \frac{8}{3}$

Since 8 and 3 are not perfect squares, $2\frac{2}{3}$ is not a perfect square.

C. $16\frac{4}{9} = \frac{148}{9}$

Since 148 is not a perfect square, $16\frac{4}{9}$ is not a perfect square.

Note: Just because 4, 9, and 16 are individually perfect squares, it does not mean $16\frac{4}{9}$ will also be a perfect square.

Part 2: Decimals

To determine whether a decimal number is a perfect square, take the square root of the number using a calculator. If either of the following conditions are met, then the decimal is a perfect square:

1. the number terminates (ends after a certain number of decimal places)

For example, 0.3 or 0.631.

2. the number repeats (has a repeating pattern of digits in the decimal)

For example, 0.33... or 0.547547547

If a decimal is NOT a perfect square, it will be non-terminating and non-repeating.

For example, 0.28564745...

Complete the following table:

Decimal	Value of Square Root	Type of Decimal	Is this decimal a Perfect Square?
1.69	1.3	Terminating	Yes
3.5	1.870828693386	Non-terminating, non-repeating	No
70.5	8.396427811873	Non-terminating, non-repeating	No
5.76	2.4	Terminating	Yes
0.25	0.5	Terminating	Yes
2.5	1.581138830084	Non-terminating, non-repeating	No
0.44...	0.666666666666	Repeating	Yes

Practice Exercises: Pg.11, # 8